Entanglement and mixedness in general probabilistic theories

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Entanglement is one of the most puzzling features of quantum theory, responsible for
many of its counter-intuitive aspects [1-3] and for many advantages in information-theoretic
protocols [4-9]. Operationally, the usefulness of entangled states can be characterized in
the LOCC paradigm [10-12], which induces a preorder on the set of bipartite states. At
the pure-state level, the preorder is completely characterized by the majorization conditions
[13-16], which set up a one-to-one correspondence between the LOCC ordering of bipartite
states and the mixedness ordering of single-party density matrices.

In this work we analyze the link between pure-state entanglement and mixedness at the
operational level, without assuming the Hilbert space formalism. To do that, we adopt
the framework of general probabilistic theories (GPT) [17-25]. We present operational
definitions for the entanglement and mixedness preorders, and, in this context, we establish
an equivalence between them based on operational axioms. The starting point to relate pure-
state entanglement to mixedness is the axiom of purification, which has been extensively
investigated in Refs. [21, 22]. This axiom allows to model every mixed state as the marginal
of a bipartite pure state in a canonical way. In every theory satisfying the purification axiom,
we show that a bipartite pure state can be LOCC-converted into another if the marginal of
the former is more mixed than the marginal of the latter.

The converse implication requires an additional assumption of symmetry, which is satis-
fied by quantum theory both on complex and real vector space. Thanks to this assumption,
we provide a new proof of the Lo-Popescu theorem [12], solely based on the GPT framework,
which leads to a full equivalence between LOCC and mixedness orderings.

Furthermore, we provide an extension of the notion of majorization to theories with
purification, and build on this result to define a class of generalized Rényi entropies. Despite
the fact that the notion of entropy in GPTs is pretty thorny [26-29], we show that under

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our axioms, these entropies share most of the features of the quantum Rényi entropies.


