Superconducting qubits

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Which quantum computer is right for you?
Which quantum computer is right for you?
There are many types to choose from. Here’s how they compare and our all-important verdict

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**New Scientist says** ⭐⭐⭐⭐⭐⭐⭐
Quantum information processing: the challenge

Qubits: Two-level systems

Two-qubit entangling gates

Qubit readout

Single-qubit control

• Conflicting requirements: long-lived quantum effects, fast control and readout

Outline

• Artificial atoms
  • Physics 101: Harmonic oscillators and basic electrical circuits
  • Superconductivity and Josephson junctions

• Circuit QED: a possible QC architecture

• Recent realizations and challenges
‘Atomic atoms’

- Control by shining laser tuned at the desired transition frequency

- Hyperfine levels of $^9\text{Be}^+$ have long relaxation and dephasing times

$$T_1 \sim \text{a few years} \quad T_2 \gtrsim 10 \text{ seconds}$$

Relaxation and dephasing times

- $T_1$: Relaxation = amplitude damping channel ≠ bit flip channel

\[ |1\rangle \quad \rightarrow \quad |0\rangle \]

\[ \text{(Energy is conserved)} \]

\[ e^{-t/T_1} = e^{-\gamma_1 t} \]

- $T_2$: Dephasing = phase damping channel = phase flip channel

\[ |\psi\rangle = c_0|0\rangle + c_1|1\rangle \rightarrow \rho = \begin{pmatrix} |c_0|^2 & c_0 c_1^* e^{-t/T_2} \\ c_0^* c_1 e^{-t/T_2} & |c_1|^2 \end{pmatrix} \]

\[ e^{-t/T_2} = e^{-\gamma_2 t} \]
‘Atomic atoms’

- Control by shining laser tuned at the desired transition frequency
- Hyperfine levels of $^9\text{Be}^+$ have long relaxation and dephasing times
  \[ T_1 \sim \text{a few years} \quad T_2 \gtrsim 10\ \text{seconds} \]
- Reasonably short gate time
  \[ T_{\text{not}} \sim 5\ \mu\text{s} \]
- Low error per gates: $\sim 0.48\%$

$E_{01} = E_1 - E_0 = \hbar \omega_{01}$

\[ \begin{align*}
|0\rangle & \quad \text{Energy} \quad |1\rangle \\
|2\rangle &
\end{align*} \]

Artificial atoms

- Based on microfabricated circuit elements
- Well defined energy levels
- Nonlinear distribution of energy levels
- Maximize numbers of thumbs up!
Avoiding dissipation: superconductivity

- Normal metals dissipate energy

- No resistance in superconducting state ⇒ no dissipation

- Superconductivity is a (macroscopic) quantum effect

- A good starting point for a quantum device...
Basic circuit elements (classical version)

**Capacitor:**
- Two metal plates separated by an insulator
- Relates voltage to charge

\[ Q = CV \]

**Inductor:**
- A non-resistive wire
- Relates voltage to change of current

\[ V = L \frac{dI}{dt} \]

\[ \Phi = LI \]

\[ \Phi = \int_{-\infty}^{t} dt' V(t') \]
Basic circuit elements *(classical version)*

**Capacitor:**
- Two metal plates separated by an insulator
- Relates voltage to charge

\[ Q = CV \]

**Inductor:**
- A non-resistive wire
- Relates voltage to change of current

\[ V = L \frac{dI}{dt} \]

**Current:**
Change of charge in time

\[ I = \frac{dQ}{dt} \]
Basic circuit elements \textit{(classical version)}

Voltage is the same across L and C:

\[ \frac{Q}{C} = L \frac{d^2 Q}{dt^2} \]

\[ Q(t) = Q(0) \cos(\omega_{LC} t) \quad \omega_{LC} = \frac{1}{\sqrt{LC}} \]

Capacitor:
- Two metal plates separated by an insulator
- Relates voltage to charge

\[ Q = CV \]

Inductor:
- A non-resistive wire
- Relates voltage to change of current

\[ V = L \frac{dI}{dt} \]

Standard toolkit

Capacitor (C)

Inductor (L)

Resistor (R)

LC oscillator
Basic circuit elements (classical version)

Oscillations of the charge:

\[ Q(t) = Q(0) \cos(\omega_{LC} t) \]

\[ \omega_{LC} = \frac{1}{\sqrt{LC}} \]

One out of countless examples of harmonic oscillator
Classical harmonic oscillator

Energy at arbitrary $x$: \[ H = \frac{p^2}{2m} + \frac{1}{2}kx^2 \]

= Hamiltonian

Frequency of oscillation: \[ \omega = \sqrt{\frac{k}{m}} \]
Quantum harmonic oscillator

Energy at arbitrary $x$:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}k\hat{x}^2$$

= Hamiltonian

Heisenberg uncertainty principle:
Impossible to know precisely both $x$ and $p$

Classical variables are promoted to hermitian operator acting on Hilbert space

$$x \rightarrow \hat{x} \quad p \rightarrow \hat{p} \quad [\hat{x}, \hat{p}] = i\hbar$$
Quantum harmonic oscillator

Energy at arbitrary $x$:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}k\hat{x}^2$$

Classical variables are promoted to hermitian operator acting on Hilbert space

$$x \rightarrow \hat{x} \quad p \rightarrow \hat{p} \quad [\hat{x}, \hat{p}] = i\hbar$$

Useful to introduce:

$$\hat{a} = \left(\frac{mk}{4\hbar^2}\right)^{1/4} \left(\hat{x} + i\frac{\hat{p}}{\sqrt{mk}}\right)$$

$$\hat{a}^\dagger = \left(\frac{mk}{4\hbar^2}\right)^{1/4} \left(\hat{x} - i\frac{\hat{p}}{\sqrt{mk}}\right)$$

Commutation relation:

$$[\hat{x}, \hat{p}] = i\hbar \rightarrow [\hat{a}, \hat{a}^\dagger] = 1$$

$$\hat{H} = \hbar\omega\hat{a}^\dagger\hat{a} = \hbar\omega\hat{n}$$

$$\omega = \sqrt{\frac{k}{m}}$$
Quantum harmonic oscillator

\[ \hat{H} = \hbar \omega \hat{a}^\dagger \hat{a} = \hbar \omega \hat{n} \quad \quad [\hat{a}, \hat{a}^\dagger] = 1 \quad \quad \hat{n} |n\rangle = n |n\rangle \]

What is the action of \( \hat{a} \) and \( \hat{a}^\dagger \) on the eigenstates of \( \hat{n} \)?

First observation: \( [\hat{n}, \hat{a}] = -\hat{a} \) and \( [\hat{n}, \hat{a}^\dagger] = \hat{a}^\dagger \)

\[ \Rightarrow \hat{n}(\hat{a} |n\rangle) = \hat{a} \hat{n} |n\rangle - \hat{a} |n\rangle = (n - 1) \hat{a} |n\rangle \]

\[ \hat{a} |n\rangle \propto |n - 1\rangle \]

Second observation: \( ||\hat{a} |n\rangle||^2 = \langle n | \hat{a}^\dagger \hat{a} | n \rangle = \langle n | \hat{n} | n \rangle = n \quad \Rightarrow \quad n \in \mathbb{N}_0 \)

\[ \hat{a} |n\rangle = \sqrt{n} |n - 1\rangle \quad \quad \hat{a}^\dagger |n\rangle = \sqrt{n + 1} |n + 1\rangle \]
Quantum harmonic oscillator

\[ \hat{H} = \hbar \omega \hat{a}^{\dagger} \hat{a} = \hbar \omega \hat{n} \]
\[ \hat{n} |n\rangle = n |n\rangle \quad n \geq 0 \]
\[ \hat{a} |n\rangle = \sqrt{n} |n - 1\rangle \]
\[ \hat{a}^{\dagger} |n\rangle = \sqrt{n + 1} |n + 1\rangle \]

\[ \hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} k\hat{x}^2 \]
\[ \omega = \sqrt{k/m} \]

Flux: \[ \Phi = \int dtV(t) \]

\[ \hat{a}^{\dagger} = \left( \frac{C}{4L\hbar^2} \right)^{1/4} \left( \hat{\Phi} - i \frac{\hat{Q}}{\sqrt{C/L}} \right) \]

\( \hat{a}^{\dagger} \) adds a photon to the LC circuit

- \( n \) = number of photons stored in the LC circuit
- Magnetic field
- Electric field

Energy

\[ \hbar \omega \]

\[ \hbar \omega \]

\[ \hbar \omega \]

Flux, \( \Phi \)

|0⟩

|1⟩

|2⟩

|3⟩
Artificial atom

- Initialization to ground state is simple

\[ \omega_{01} = \frac{1}{\sqrt{LC}} \approx 10 \text{ GHz} \]
\[ \approx 0.5 \text{ K} \]

- Not a good «two-level» atom, not a qubit…
Josephson junction

Josephson junctions

Superconductor (Al)
Insulator (AlO_x)
Superconductor (Al)

Standard toolkit

Capacitor (C)
Inductor (L)
Resistor (R)
Josephson junctions

Standard toolkit:
- Capacitor (C)
- Inductor (L)
- Resistor (R)

Josephson junctions:

Scale: 100 nm
Artificial atom toolkit

Capacitor (C):
- Two metal plates separated by an insulator
- Relates voltage to charge

\[ Q = CV \]

Inductor (L):
- A non-resistive wire
- Relates current to flux

\[ V = L \frac{dI}{dt} \]

Josephson junction:
- Two superconductors separated by an insulator
- Relates current to flux

\[ \Phi = L I \]
\[ \Phi = \int_{-\infty}^{t} dt' V(t') \]

\[ I = I_0 \sin(2\pi \Phi / \Phi_0) \]
Superconducting artificial atom

- Very short $\pi$-pulse time
  \[ T_\pi \sim 4 - 20 \text{ ns} \]
- Big improvements in relaxation and dephasing times in last 10 years
- Error per gates of 0.2%, similar to trapped ion results

\[
V(t) = V_0 \cos \omega_0 t
\]

Superconducting *transmon* qubits
Superconducting qubits, a family tree

**Charge**
- NEC, Saclay, 1999

**Phase**
- NIST 2002

**Flux**
- Delft, 1999

**Quantronium**
- Saclay, 2002

**Transmon**
- Yale, 2007

**xmon**
- UCSB, 2013

**Fluxonium**
- Yale, 2009
Circuit QED

\[ \hat{V} = V_0 \cos(\omega t) \]
Circuit QED

\[ V(t) = V_0 \cos \omega_0 t \]
Circuit QED: Multi-qubit architecture

\[ V(t) = V_0 \cos(\omega_0 t) \]

\[ V(t) = V_0 \cos(\omega_0 t) \]
Circuit QED: Resonant and dispersive regimes

**Resonant regime:**
- Identical 0-1 transition frequencies
- Energy exchange between qubits and oscillator
- Oscillator acts as quantum bus for entangling qubits

**Dispersive regime:**
- Different 0-1 transition frequencies
- No energy exchange between qubits and oscillator
- Qubit-state dependent oscillator frequency leads allows qubit readout
Circuit QED: Multi-qubit architecture

Quantum bus: entangling gates and readout

Single-qubit control
Circuit QED: ‘1D’ realization

Circuit QED: ‘1D’ realization

Circuit QED: scaling up
Circuit QED: Multi-qubit architecture

\[ V(t) = V_0 \cos \omega_0 t \]

Long-range qubit-qubit interactions
Circuit QED: alternative architecture

\[ V(t) = V_0 \cos \omega_0 t \]

Individual qubit readout

Two-qubit gates

Short-range qubit-qubit interactions
Circuit QED: recent realizations and challenges
10 years of circuit QED


Quantum optics on a chip with artificial atoms
Quantum information processing
Past, present and future

- Operation on single physical qubits
- Algorithms on multiple physical qubits
- QND measurement for error correction and control
- Logical memory with longer lifetime than physical qubits
- Operations on single logical qubits
- Algorithms on multiple logical qubits
- Fault-tolerant quantum computation

Complexity

Time

Superconducting qubits
Trapped ions
Rydberg atoms
Spin qubits

High-fidelity gates and readout

### Gates

#### Single-qubit gate
- Average fidelity: > 99.92%
- Error when simultaneously operating neighbour qubits: < $10^{-4}$
  - UCSB: Nature 508, 500 (2014)

#### Two-qubit gate (direct)
- Average fidelity: up to 99.4%
  - UCSB: Nature 508, 500 (2014)

#### Two-qubit gate (via bus)
- Average fidelity: > 96.75%
  - IBM: PRL 109, 060501 (2012)

### Readout

#### Single-qubit readout
- Fidelity: up to 99.8% in 140 ns
  - UCSB: PRL 112, 190504 (2014)

#### Two-qubit readout (logical basis)
- Fidelity: > 90%

#### Two-qubit readout (Bell basis)
- Bell state concurrence: ~ 35%
  - UCB: PRL 112, 170501 (2014)

### Complexity

#### Max number of qubits and resonators
- Direct: 9 qubits and 10 resonators
  - UCSB: 1411.7403
- Bus: 5 qubits and 7 resonators
  - Delft: 1411.5542
Recent realizations

**Simple algorithms**

**Deutsch–Jozsa**

**Grover (N=4)**
Saclay: PRB **85**, 140503 (2012)

**QFT**
UCBS: Science **334**, 61 (2011)

**Shor (15; compiled)**

**Protocols**

**Deterministic teleportation**
ETH Zurich: Nature **500**, 319 (2013)

**Quantum error correction**

**3-qubit code**
Yale: Nature **482**, 382 (2011)
Recent realizations

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UCBS: Nature Physics 8, 719 (2012)

**Protocols**

**Deterministic teleportation**
ETH Zurich: Nature 500, 319 (2013)

**Quantum error correction**

**3-qubit code**
Yale: Nature 482, 382 (2011)

Error detection via parity meas.
« ... using a two-by-two lattice of superconducting qubits to perform syndrome extraction and arbitrary error detection via simultaneous quantum non-demolition stabilizer measurements. This lattice represents a primitive tile for the surface code ... »

IBM: 1410.6419
Delt: 1411.5542
Recent realizations

Simple algorithms

Deutsch–Jozsa

Grover (N=4)
Saclay: PRB 85, 140503 (2012)

QFT
UCBS: Science 334, 61 (2011)

Shor (15; compiled)
UCBS: Nature Physics 8, 719 (2012)

Protocols

Deterministic teleportation
ETH Zurich: Nature 500, 319 (2013)

Quantum error correction

Error detection via parity meas.

«... we report the protection of classical states from environmental bit-flip errors and demonstrate the suppression of these errors with increasing system size.»

Data qubit avg.
5 qubit R.C.
9 qubit R.C.

Probability, Fidelity

Total repetition code cycles - k

UCSB: 1411.7403
Past, present and future

- Operation on single physical qubits
- Algorithms on multiple physical qubits
- QND measurement for error correction and control
- Logical memory with longer lifetime than physical qubits
- Operations on single logical qubits
- Algorithms on multiple logical qubits
- Fault-tolerant quantum computation

Complexity

Time
Under the rug…
Summary

- Artificial atoms based on Josephson junctions
  - Low error per gate
  - Steady improvement
- Circuit QED
  - Resonator acts as bus for entangling gates
  - Dispersive regime: high-fidelity qubit readout
- Basic protocols being implemented
Superconducting qubits and circuit QED
Quantum error correction, quantum algorithms, ...
Quantum dots, NV centers, ...
Full counting statistics and quantum noise

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