Topological Order via Matrix Product Operators

Burak Sahinoglu

University of Vienna

D. Williamson (Vienna), N. Bultinck, M. Marien, J. Haegeman (Ghent), N. Schuch (Aachen), F. Verstraete (Vienna-Ghent)

merged with

Matrix Product Operators: Local Equivalence and Topological Order

Oliver Buerschaeper

Perimeter Institute – FU Berlin

This talk is **NOT**

- A condensed matter talk

  - no approximations

  - no correlation functions, etc.

- A quantum information theory talk

  - no channel (capacity)

  - no asymptotic (or one-shot) quantitiy

This talk is about special states with certain type of entanglement

- Ground state spaces
  - of many-body lattice models
  - which are long-range entangled
  - and topology dependent

OUTLINE

- Motivations

  - Quantum Error Correcting Codes

  - Material vs. Order

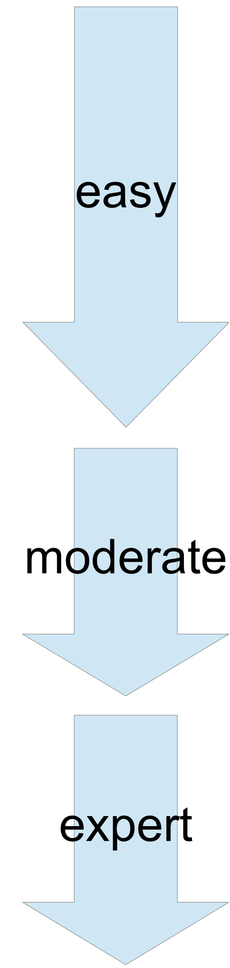
- A natural tool: Tensor Network States (TNS)

  - Topological order in TNS

  - Examples: Twisted Quantum Doubles

    String-net condensed states

- Future



Quantum Error Correcting Codes

- Kitaev:

  - Encode the logical qubits in topological data so that local noise cannot change the logical qubit.

  - Any nontrivial operation inside of the codespace must be topologically nontrivial → local noise leads to an error with infinitesimal probability.

- Example: Toric Code

  - 2 qubits on torus (g qubits on g-genus surface)

  - Wilson loops as operations on codespace.

Material vs. Order

- Whole from elementary:

  Electrons, protons, etc..

How diversity emerges from elementary parts?

Order → Diversity

Phases of Quantum Matter

- Classical systems: Frozen at T=0.
- Quantum systems with local order parameter
- Quantum systems with nonlocal order parameter

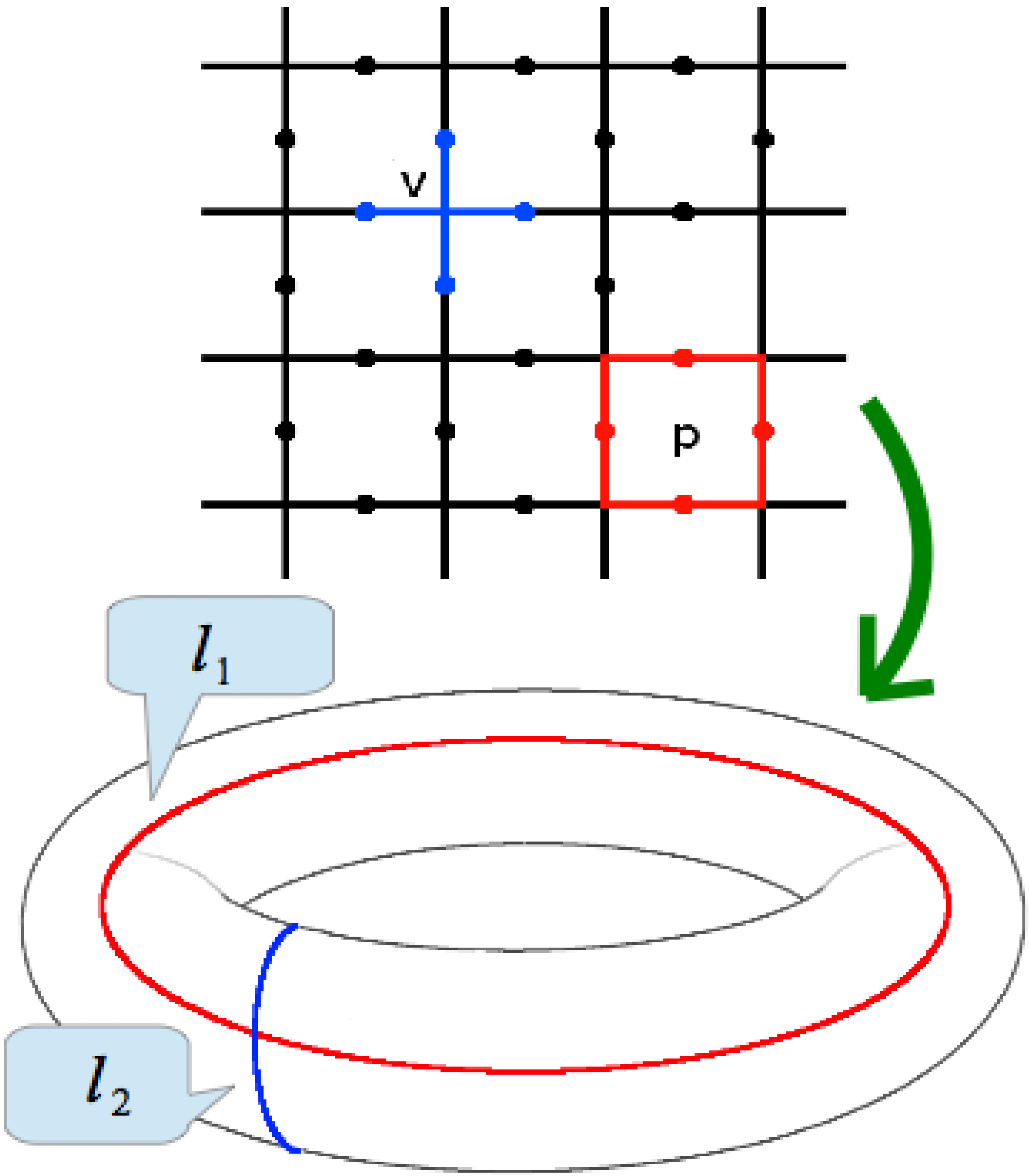
Topology dependent ground states

Local indistinguishability

This talk is about special states with certain type of entanglement

- Ground state spaces
  - of many-body lattice models
  - which are long-range entangled
  - and topology dependent

Example: Toric Code



$$A_v = \Pi_{i \in v} X_i \,,\, B_p = \Pi_{i \in p} Z_i$$

$$H = -\Sigma_v A_v - \Sigma_p B_p$$

Ground state space:

$$|\psi_1\rangle = \Sigma |even - l_1 \wedge even - l_2\rangle$$

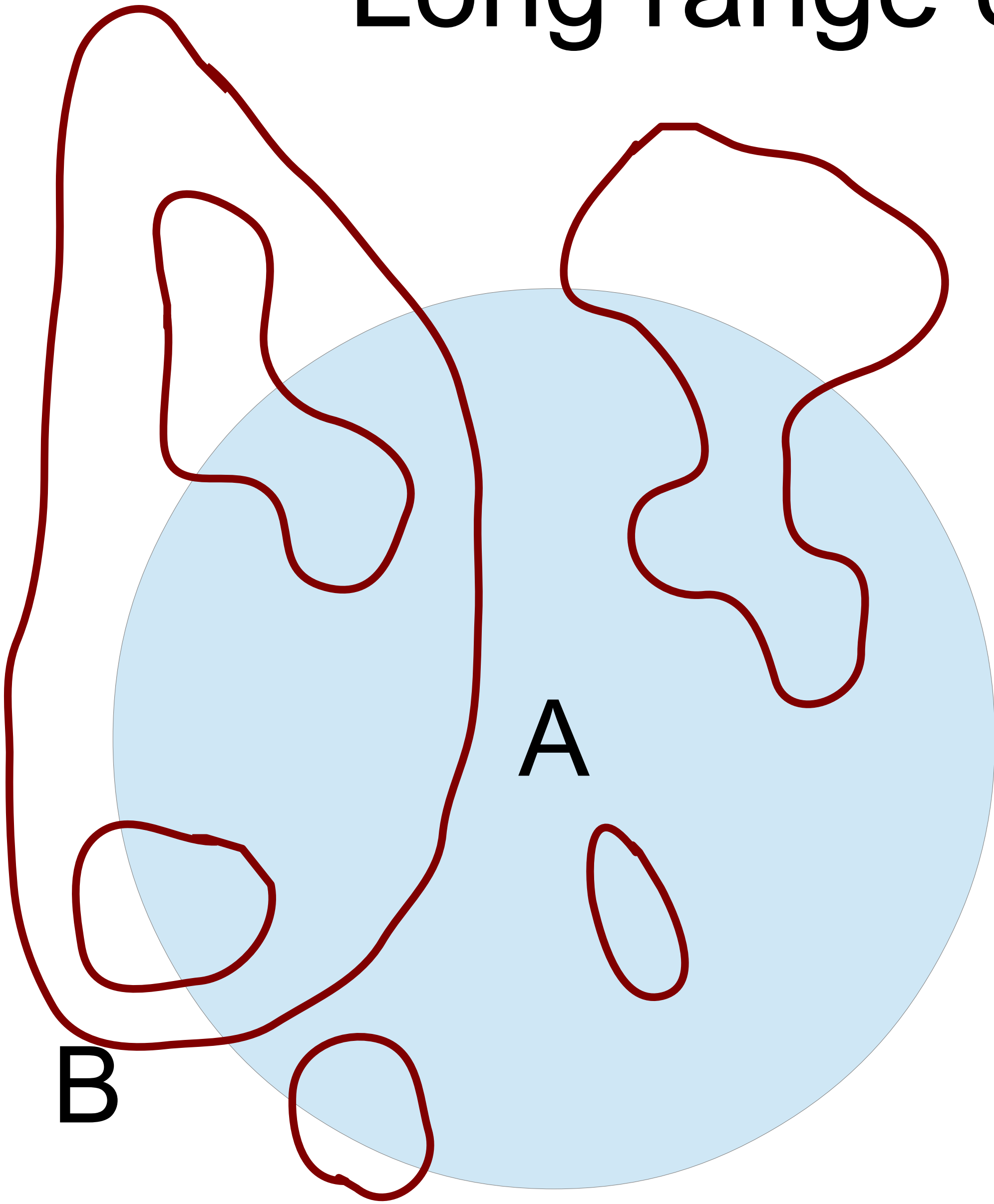
$$|\psi_2\rangle = \Sigma |even - l_1 \wedge odd - l_2\rangle$$

$$|\psi_3\rangle = \Sigma |odd - l_1 \wedge even - l_2\rangle$$

$$|\psi_4\rangle = \Sigma |odd - l_1 \wedge odd - l_2\rangle$$

Locally Indistinguishable!

Long range entanglement

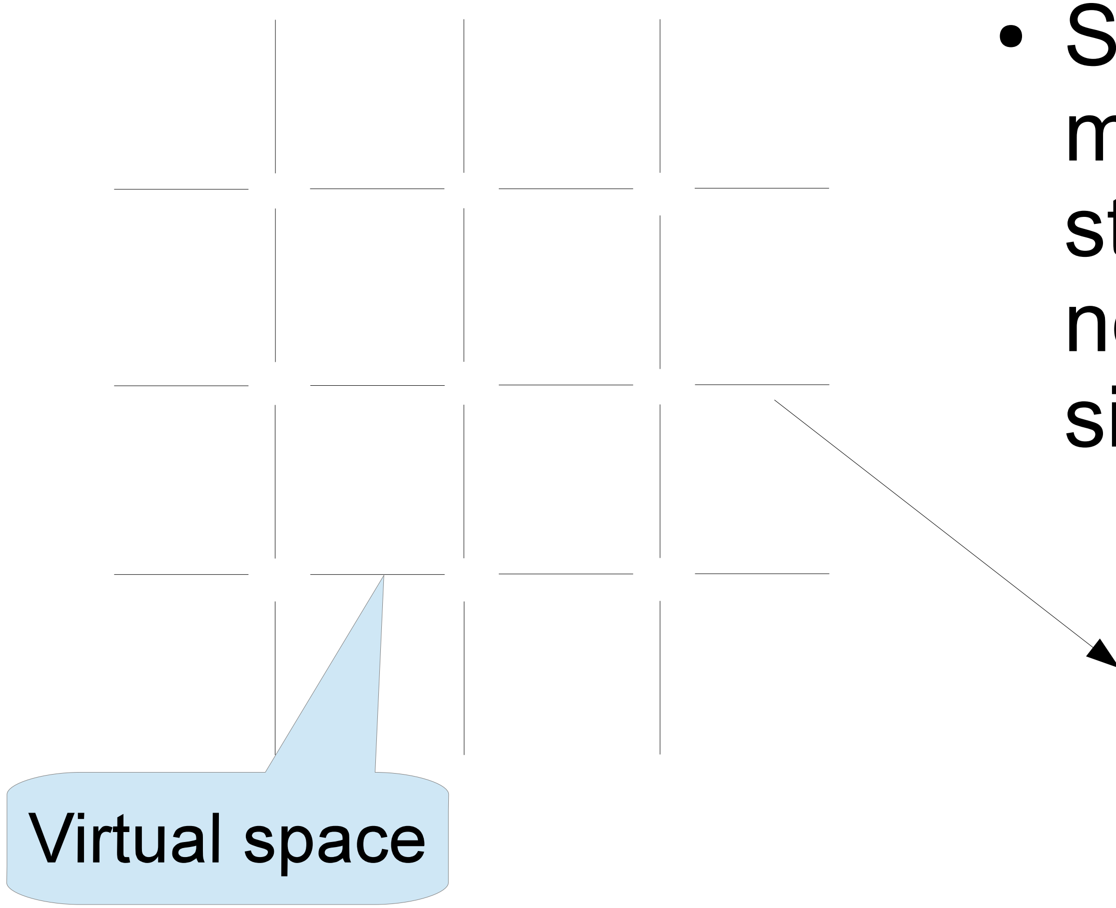


- #1s passing through the boundary= Even
- Correction to area law:

$$S(A) = L(A) - \gamma$$

Topological Entanglement Entropy

A natural tool: Tensor network states

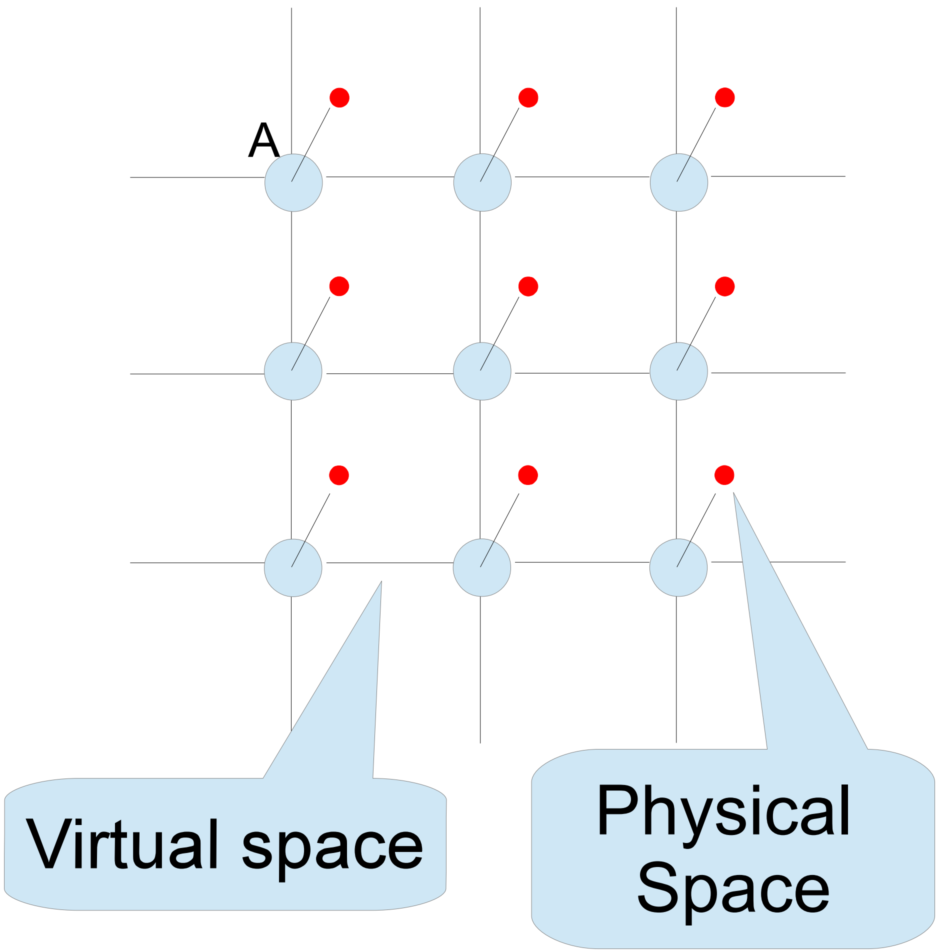


- Start with bipartite maximally entangled states between each nearest neighbour site:

$$\omega = \Sigma_{i=1}^{D} |i\rangle |i\rangle$$

$$\Psi' = \omega^{\otimes N} \quad \Rightarrow \quad H' = \Sigma_i \left(I - |\omega\rangle\langle\omega| \right)_i$$

A natural tool: Tensor network states



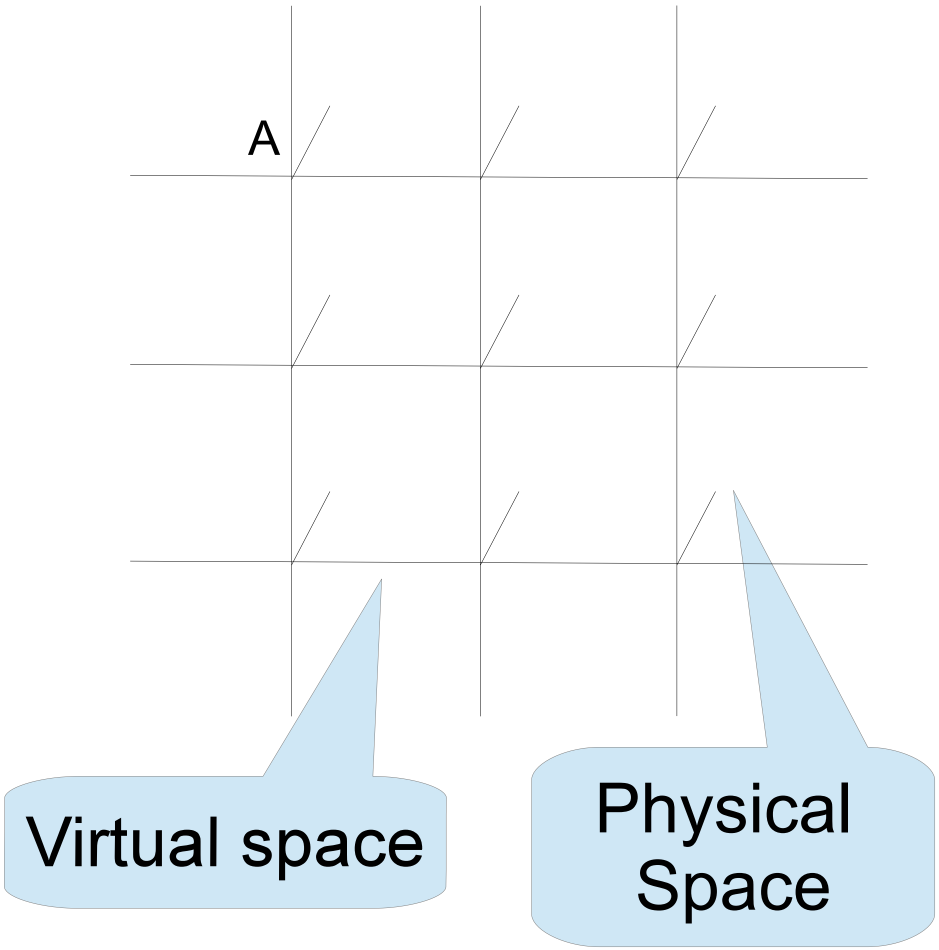
- Insert a linear map at every site:

$$A : Virtual \rightarrow Physical$$

$$\Psi = A^{\otimes N} \omega^{\otimes N}$$

$$H = A^{\otimes N} H' \left(A^{-1} \right)^{\otimes N}$$

A natural tool: Tensor network states



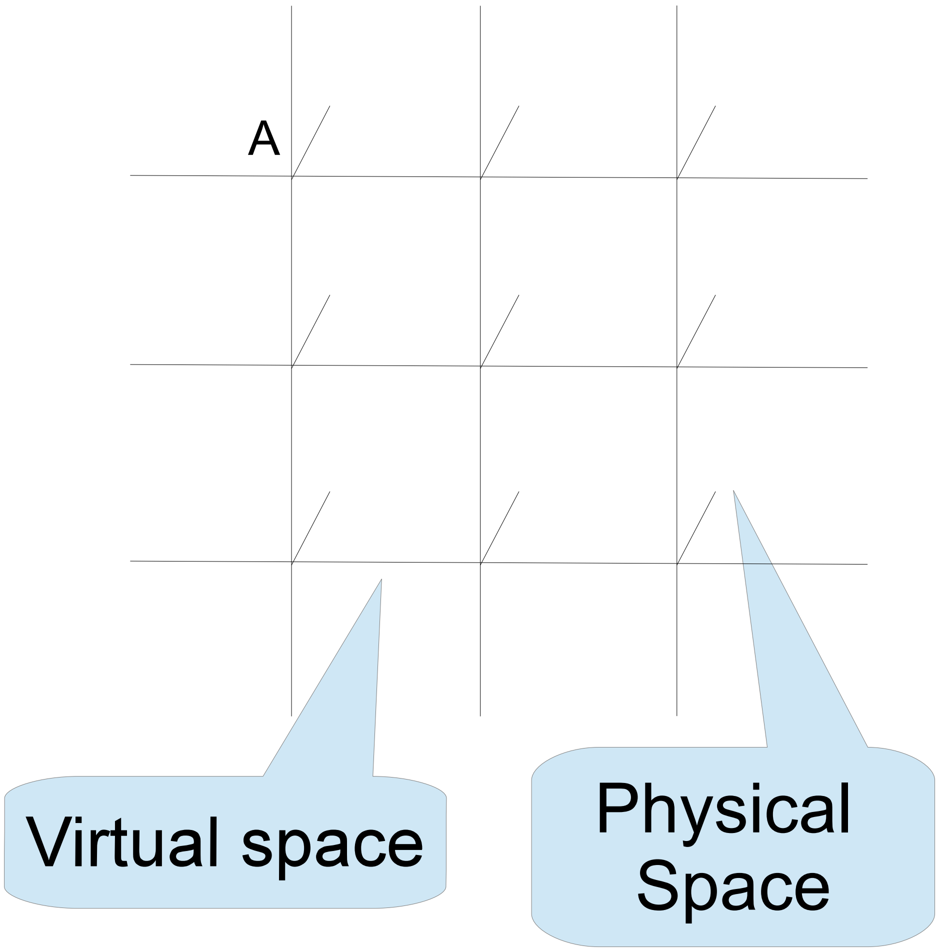
- Insert a linear map at every site:

$$A : Virtual \rightarrow Physical$$

$$\Psi = A^{\otimes N} \omega^{\otimes N}$$

$$H = A^{\otimes N} H' \left(A^{-1} \right)^{\otimes N}$$

Pedagocigal Summary of TNS



- There are virtual and physical Hilbert spaces
- The structure of the whole state is encoded in

  A (local tensor)
- Local tensor → State

  State → Local Hamiltonian
- Numerous other properties about entanglement entropy, efficient simulation of quantum systems, etc..

Topological order in TNS

- Aims:

  - Define properties of local tensor such that topological order emerges in TNS.

  - Explain nonRG-fixed point topologically ordered models.

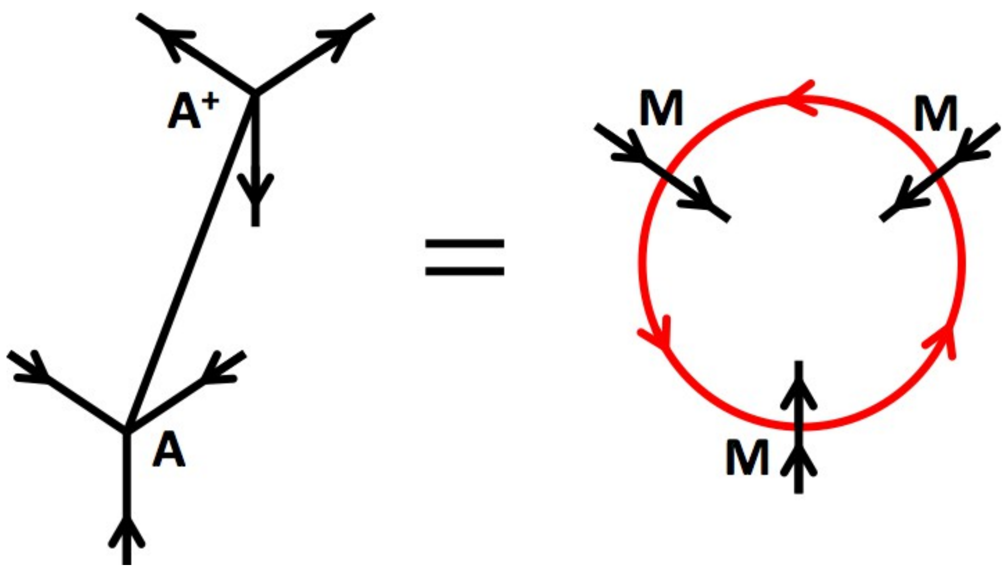
  - Find new models.

- New concepts:

  - Express local virtual subspaces in terms of Matrix Product Operators (MPO-injectivity)

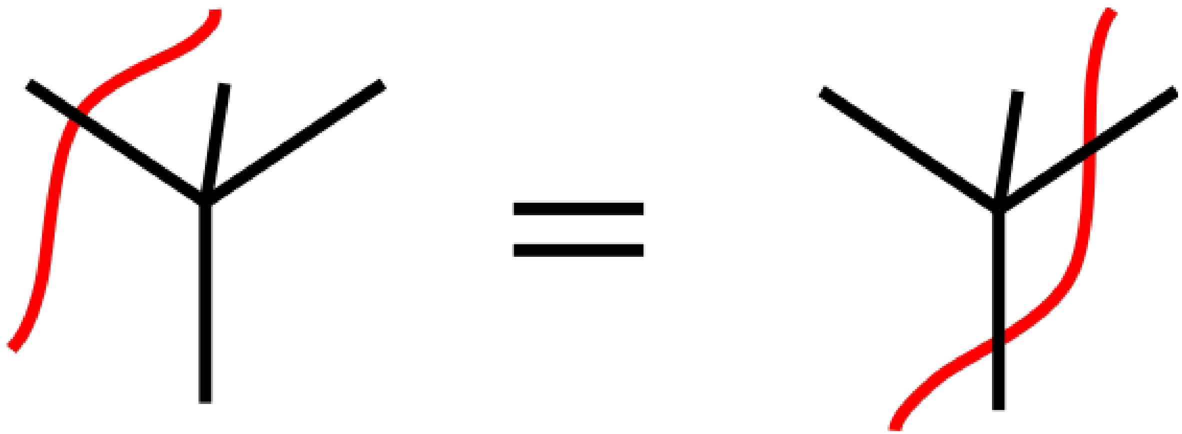
  - Symmetries of local tensor (Pulling through)

Defining the local subspace: MPO injectivity



- The virtual degrees of freedom are accessible in a subspace determined by a closed loop of MPOs.

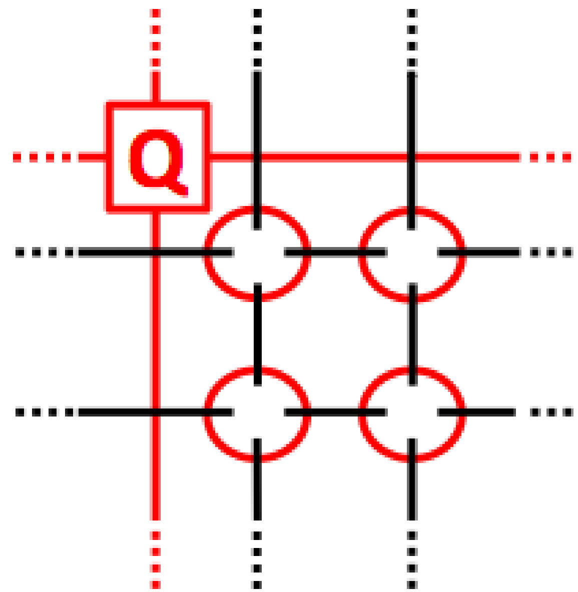
The symmetry on the virtual level: Pulling through



- Except end points, MPOs are free to move on the lattice: No change in the state!

  (Analogue of deforming Wilson lines)

Ground states



- Ground states are determined by tensor Q!
- The place of Q is irrelevant

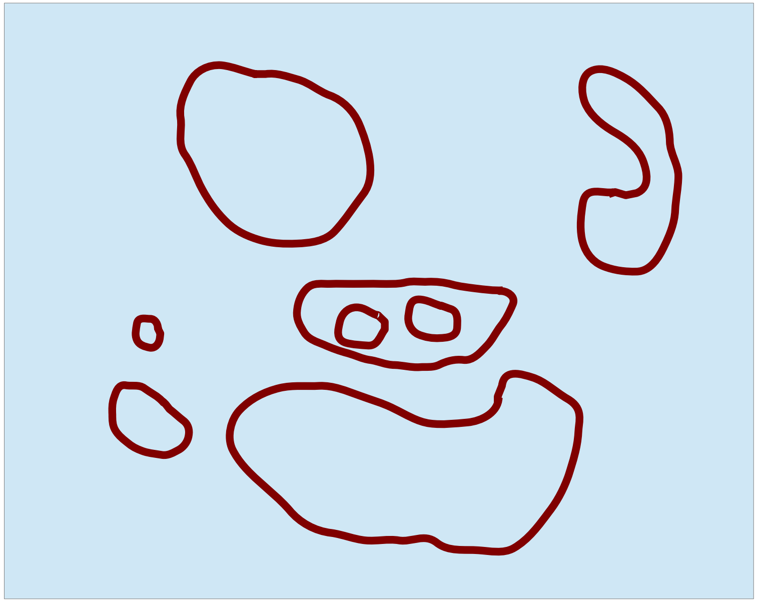
→ Find linearly independent states.

Examples

- Twisted Quantum Doubles

- String-net states

Twisting the Toric Code



- Toric code ground state

$$\Psi_+ = \Sigma \left| loops \right\rangle$$

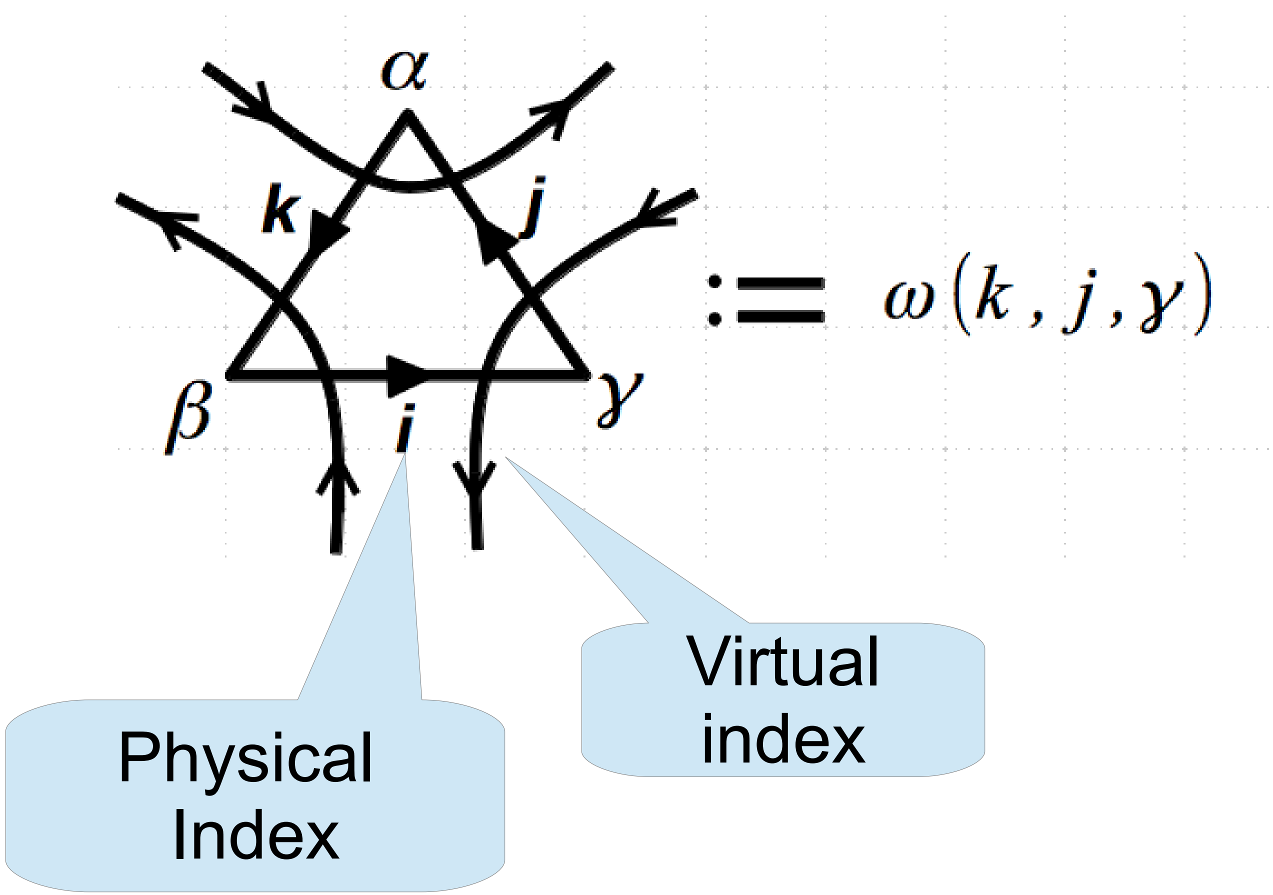
- Doubled Semion ground state

$$\Psi_- = \Sigma (-1)^{\# loops} \left| loops \right\rangle$$

Twisted Quantum Doubles

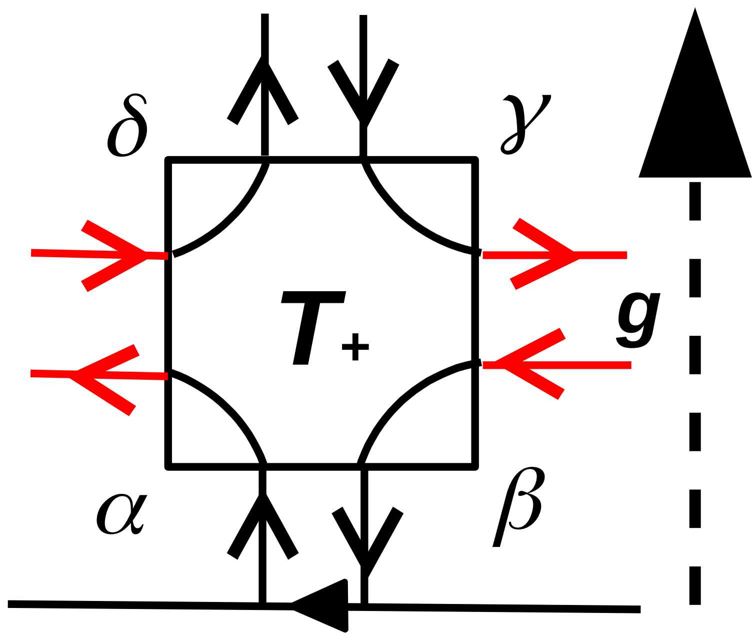
$$\omega : G \times G \times G \rightarrow U(1)$$

Special phases depending on the group element

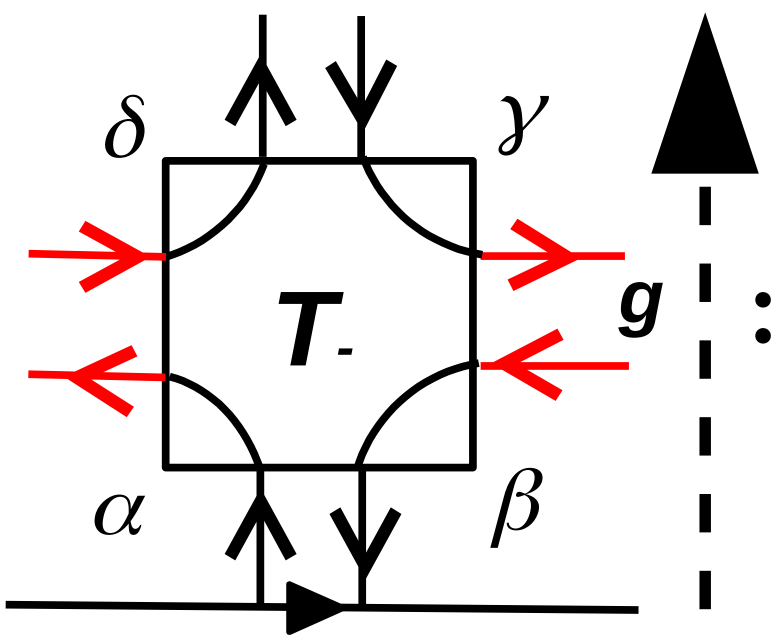


Physical indices are uniquely determined from virtual indices, via group operation!

MPOs for Twisted Quantum Doubles

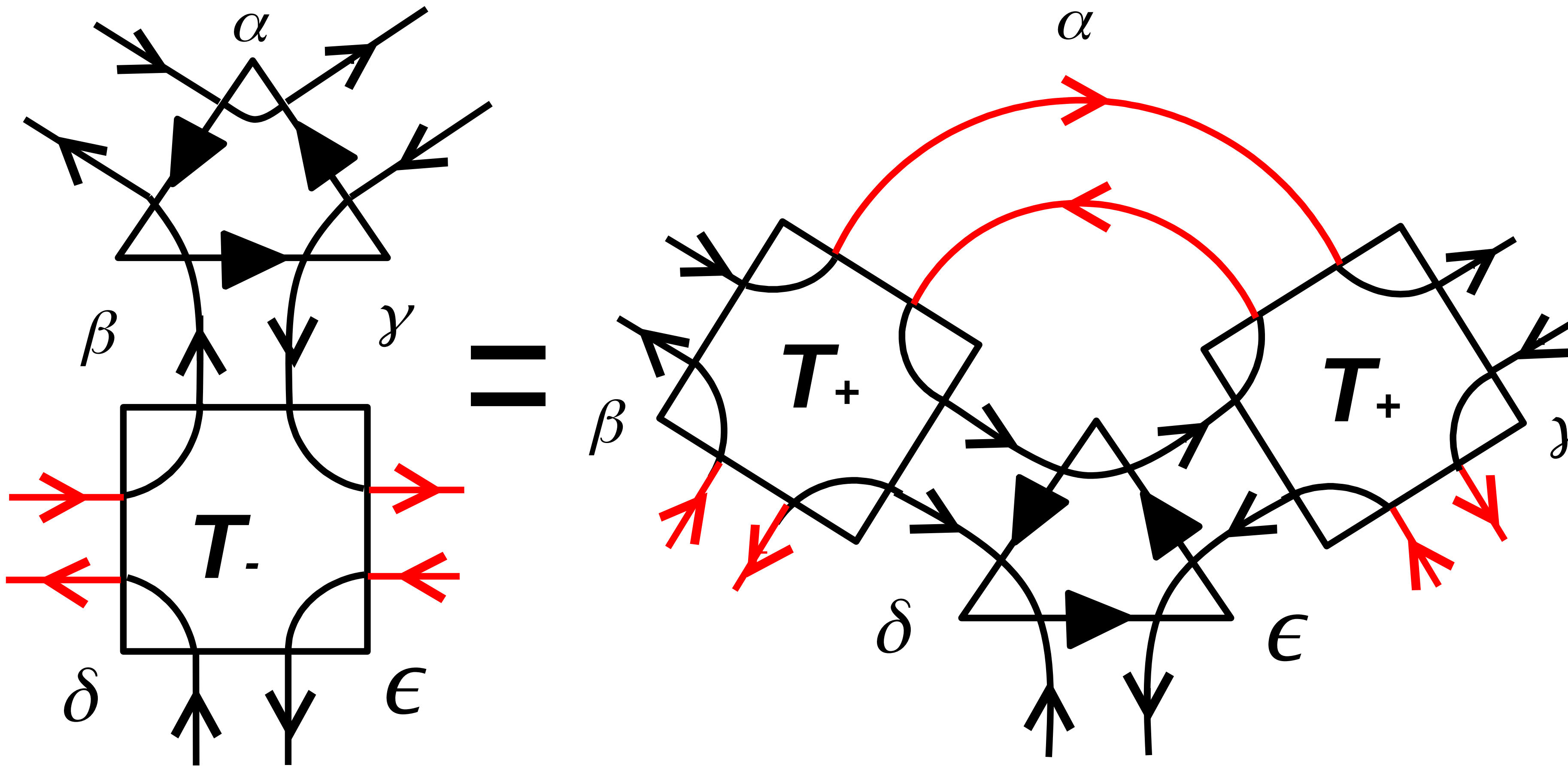


$$:= \delta_{\alpha^{-1}\delta, g}\,\delta_{\beta^{-1}\gamma, g}\,\omega(\alpha\beta^{-1}, \beta, g)^{-1}$$

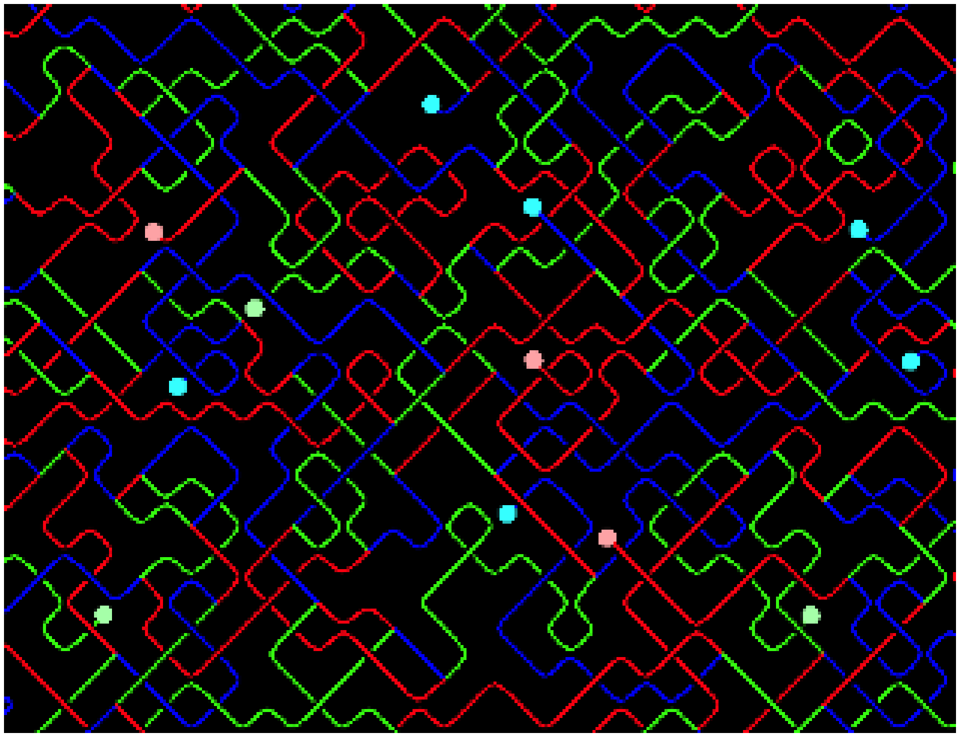


$$:= \delta_{\alpha^{-1}\delta, g}\,\delta_{\beta^{-1}\gamma, g}\,\omega(\beta\alpha^{-1}, \alpha, g)$$

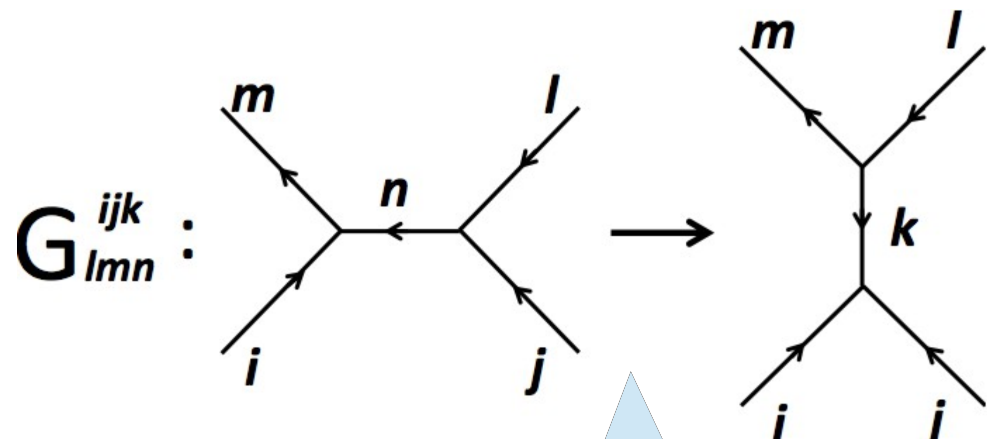
Pulling through for Twisted Q. Doubles



Levin-Wen Models: String-nets



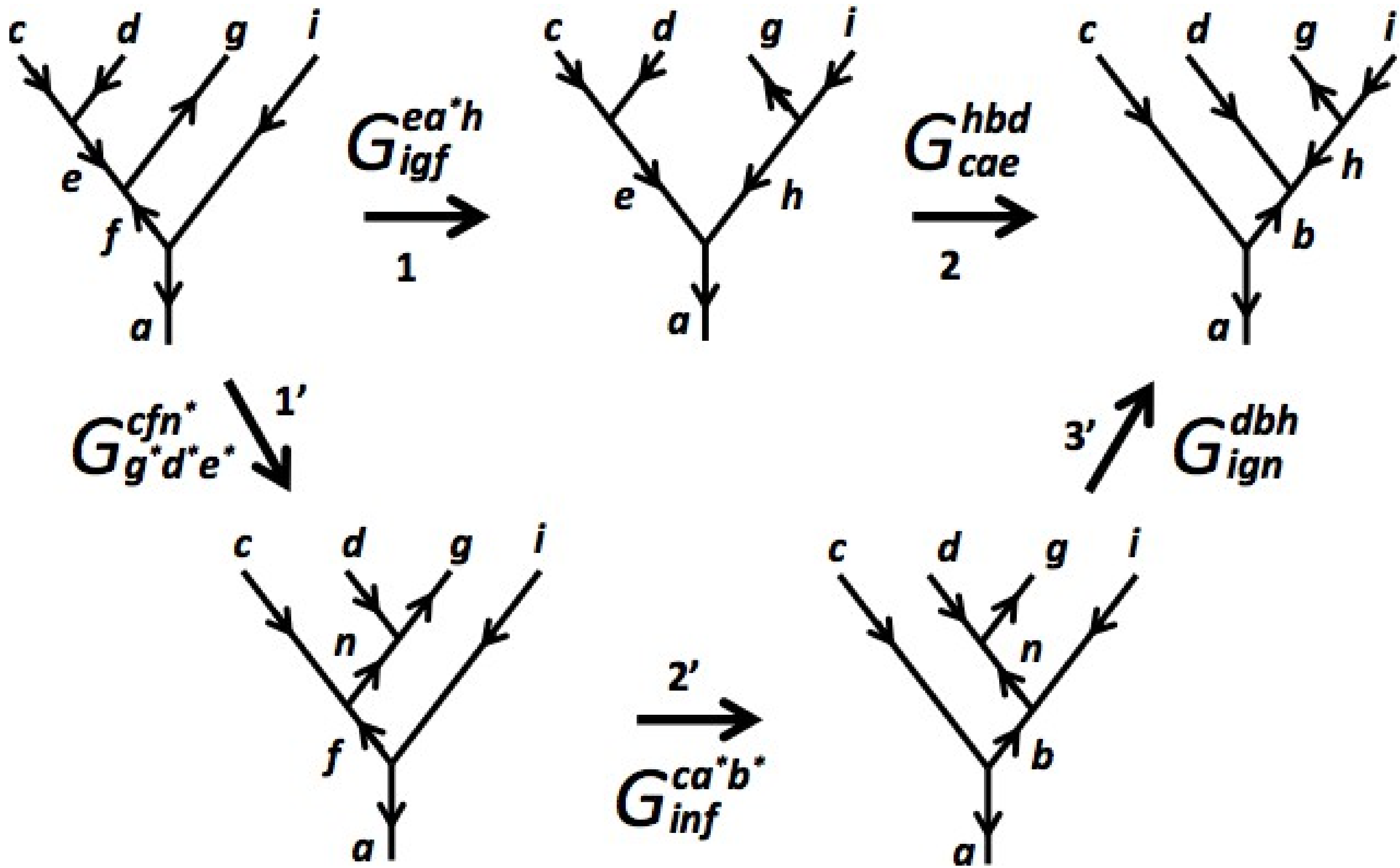
- Moving strings is free!
- Trivial loops are free
- Additional local rule:



Superposition of strings on the lattice

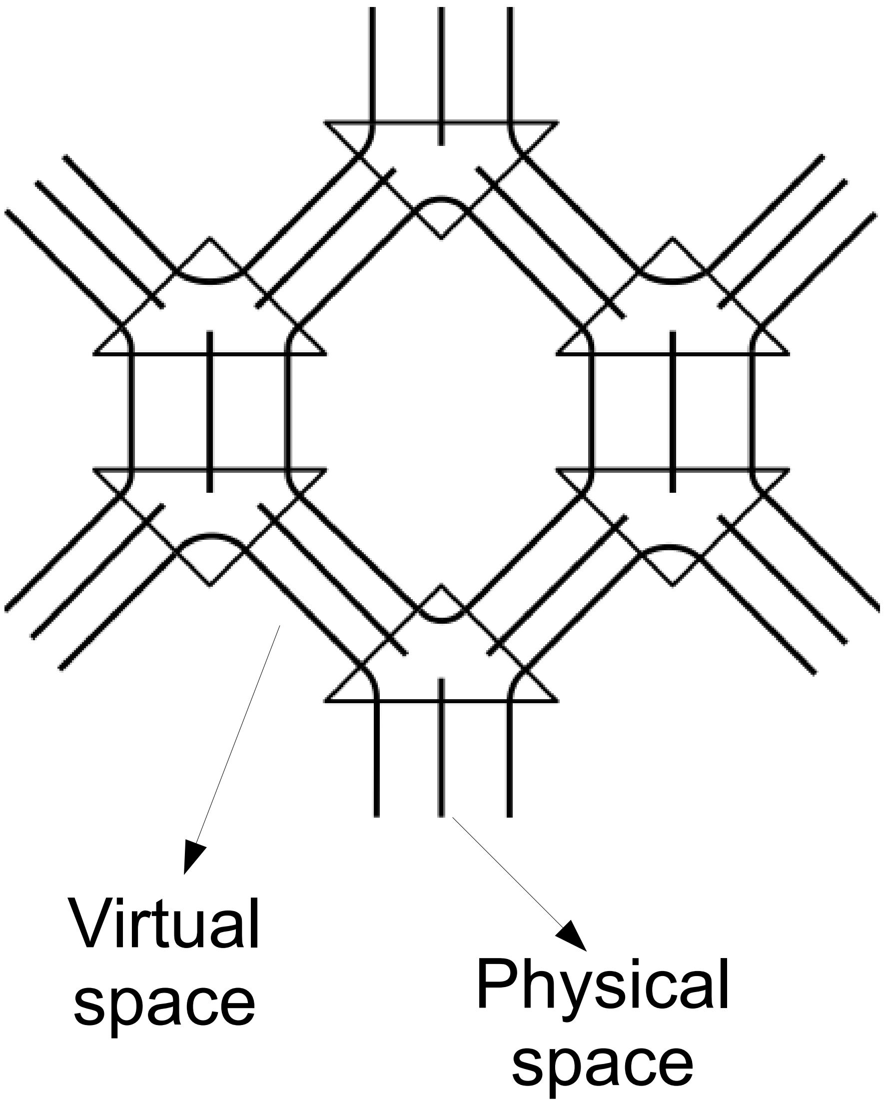
G-symbol

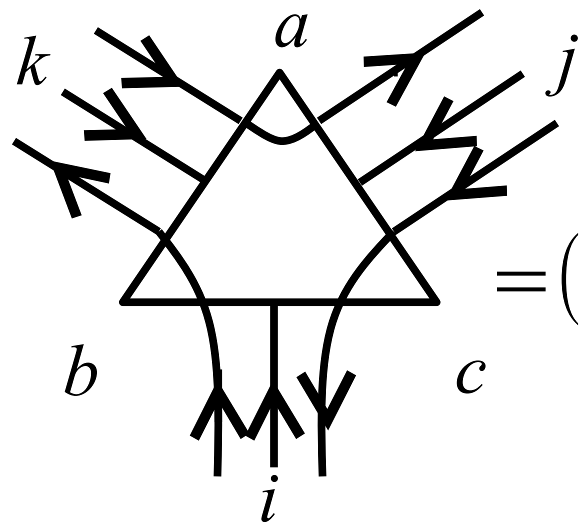
Pentagon equation (coherence condition for ground states)



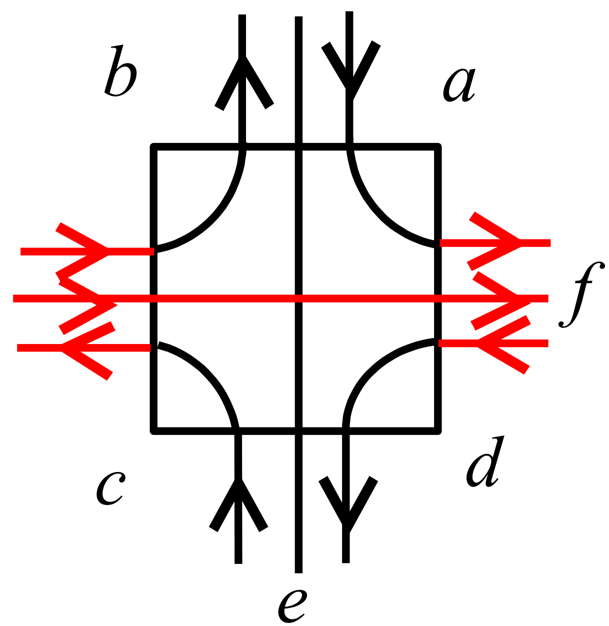
A TNS picture of String-Nets

Buerschaper, Aguado, Vidal - 2008
Gu, Levin, Swingle, Wen - 2008



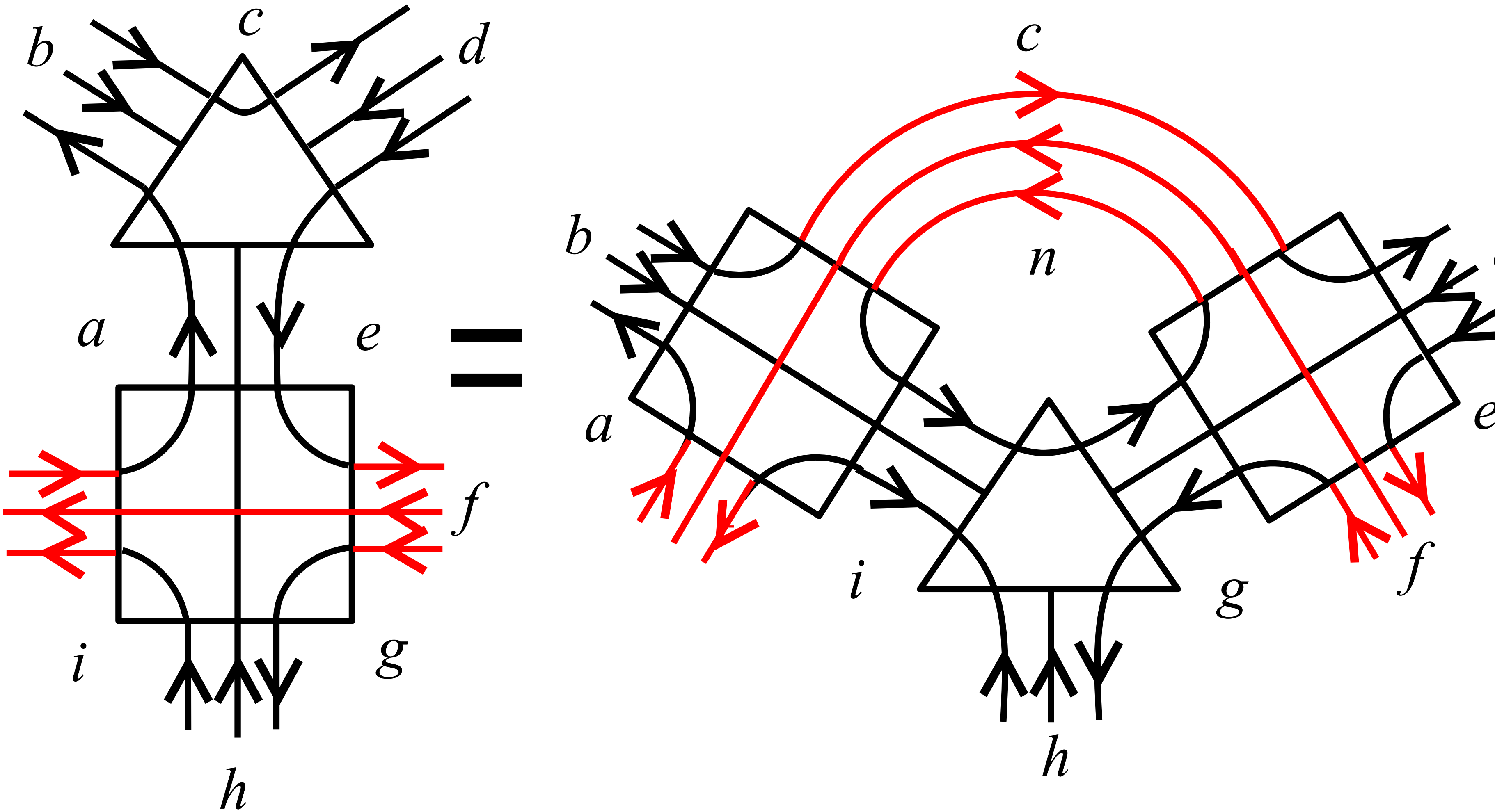


$$= \left(v_i v_j v_k\right)^{1/2} G^{ijk}_{abc}$$



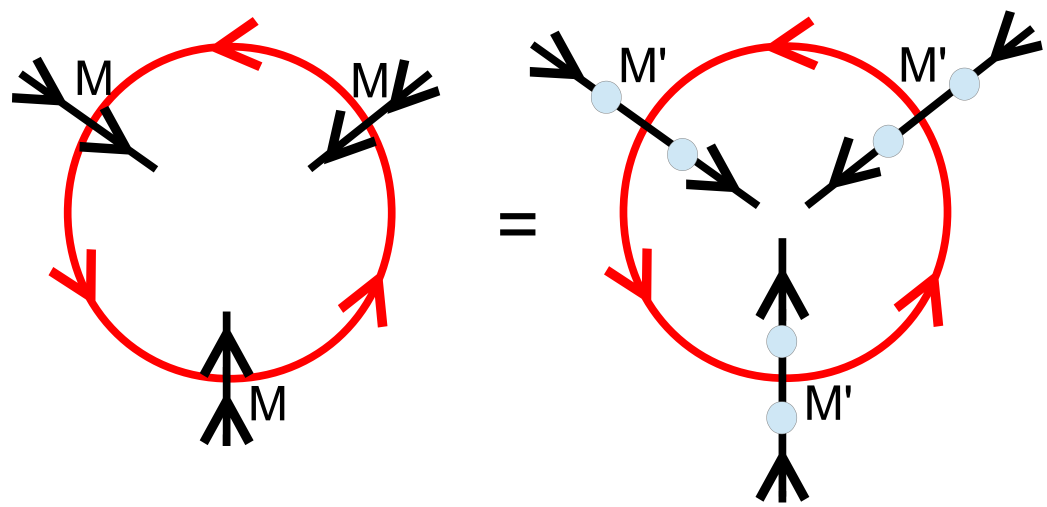
$$= G^{ab^*e}_{cdf}$$

Pulling through for String-nets



Classification of MPOs

M ~ M' if



- Trivial: product of diagonals

  e.g.: $\omega' = \omega(\phi\,\phi)/(\phi\,\phi)$ → group cohomology

- Product of unitaries

  e.g.: group aut. → group coh. collapses

- MPOs...

Morita equivalence

Summary

- Quantum error correcting codes ↔ Phases of matter
- Tensor networks states as a natural tool for studying ground states of physical systems
- Axioms for topological order (non RG-fixed point):

  - MPO-injectivity

  - Pulling through
- Layers of local equivalence

Future

- Classification:

  - Excitations

  - Topological phase transitions

- Easy to give string tension and study anyon condensation:

arXiv:1410.5443

- New models:

  - in 2D

  - Axioms generalize to higher dim.

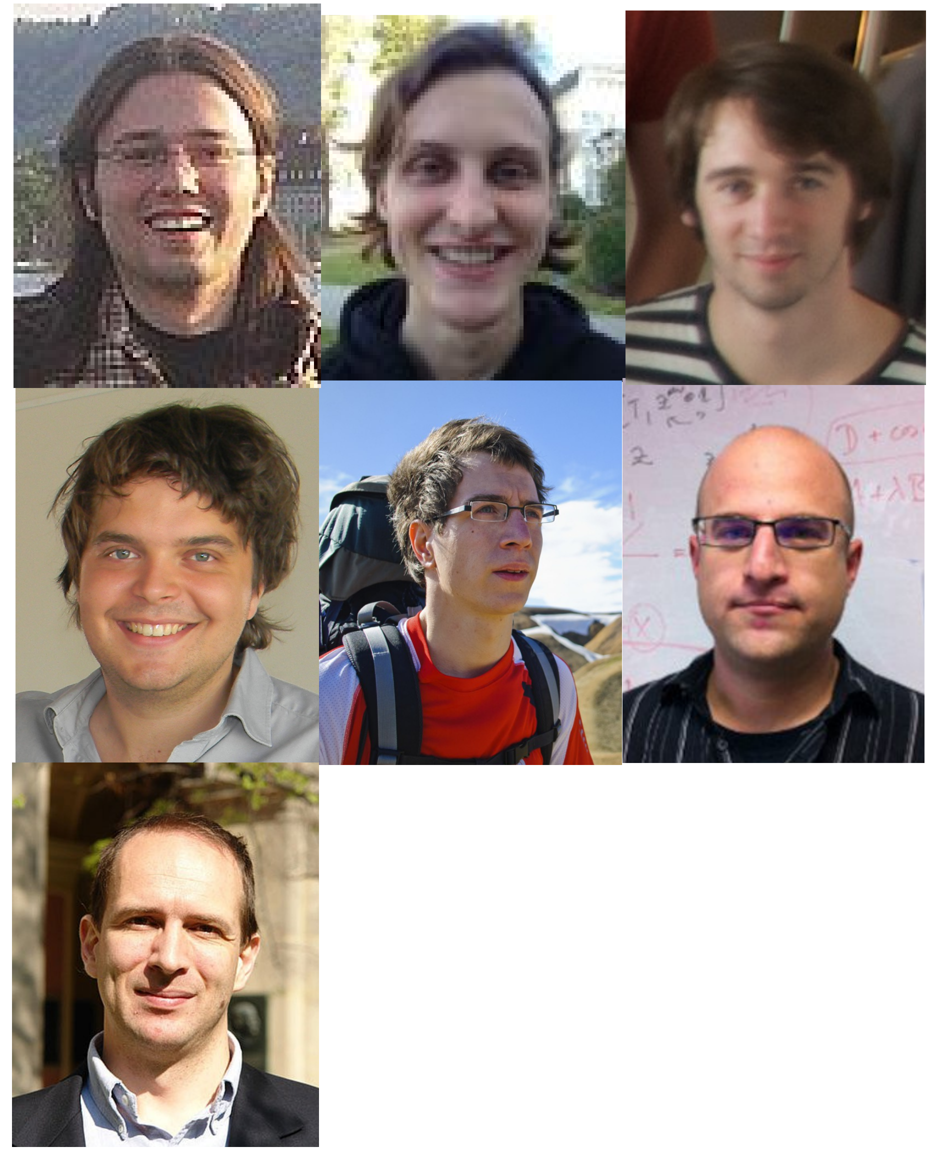
  - Haah's code etc. (?)

- Duality in PEPS:

SPT – Topological phase duality:

arXiv:1412.5604

For Details



–Ann. Phys. 351, 447–476 (2014)

– arXiv: to appear

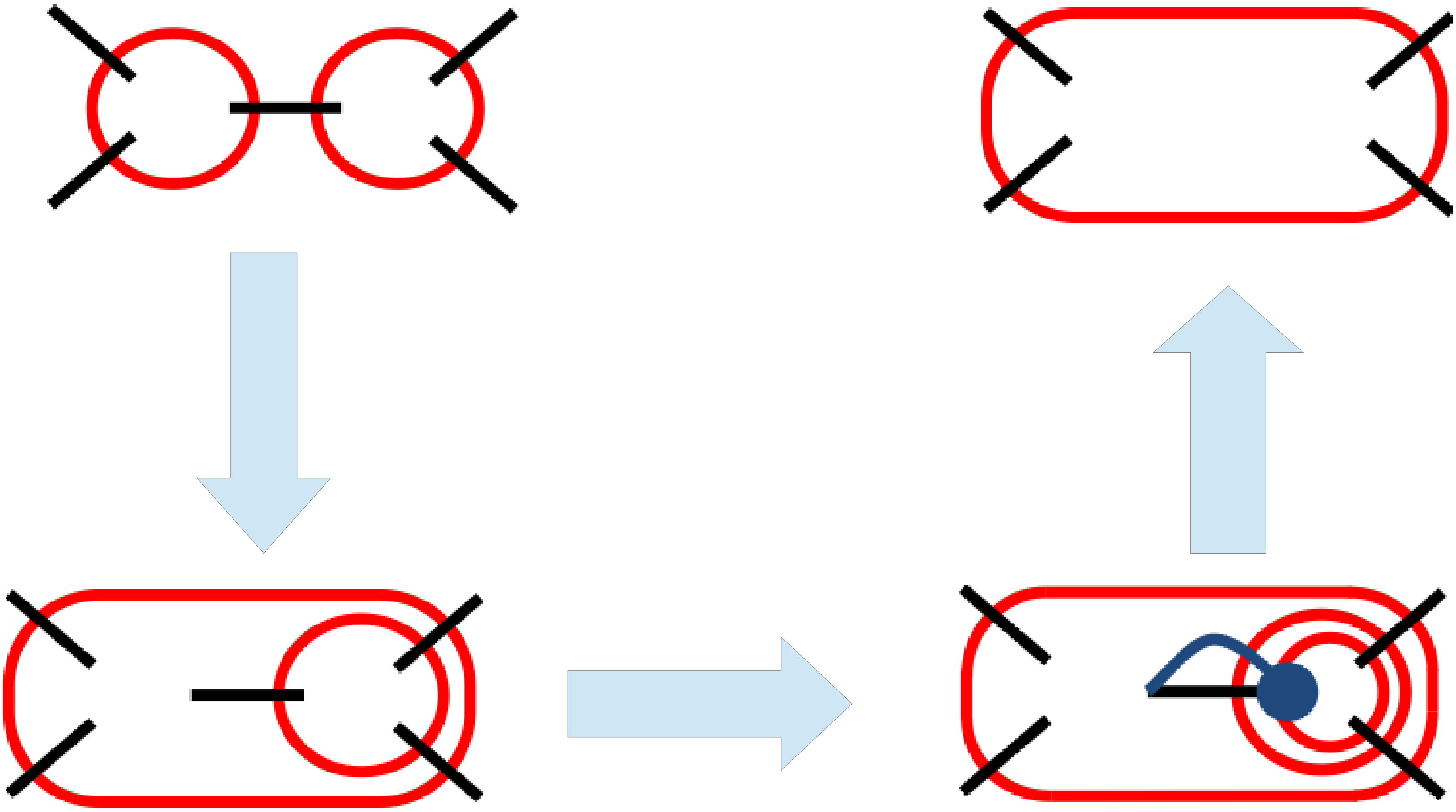
– arXiv:1409.2150

Tensor Network Summer School:

- June 1-5, 2015
  Ghent, Belgium
- Aspects of tensor networks:
  MPS, PEPS, MERA
- Check:
  www.tnss.ugent.be
  for more info.

Technical properties - 1

- Concatenation:



Technical properties - 2

- Intersection