On the informational completeness of local observables : extended abstract

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(Dated: November 24, 2014)

One of the fundamental obstacles in studying quantum many-body systems lies on the exponential blowup of the underlying Hilbert space dimension. Indeed, this "curse of dimensionality" makes many approaches based on exact numerical methods to be quite limited. Physicists have been dealing with this problem by either studying the effective field theory that is believed to describe the universal properties of the system,[1] or by studying exactly solvable models and focusing on the properties that remains stable under a generic perturbation.[2–4] These approaches are useful for predicting and identifying the properties of the system that are robust against experimental imperfections.

A recent progress on experimentally preparing many-body quantum states in the laboratory has led to a number of new theoretical questions.[5] These systems allow high-precision controls over each individual degrees of freedom, which makes them as a viable candidate for the quantum simulator.[6, 7] Quantum simulator, first envisioned by Feynmann, is a quantum machine that can simulate any local quantum system. Using such a device, one would be able to simulate the dynamics of many-body systems, a task that seems to be intractable with the existing classical computers. One of the important questions regarding these systems is whether one can verify that such a machine is performing a desired task.

In particular, a fundamental question is whether one can verify, in a scalable manner, that the prepared state in the laboratory is indeed close to the state that the experimentalist intended to prepare. Such a task can be performed in a time that scales polynomially with the system size for a large class of interesting states, such as stabilizer states and generic matrix product states.[8–10] However, the case remained open for higher-dimensional systems.

The primary purpose of this paper is to show that such a task can be achieved for two-dimensional(2D) systems as well; it can be performed in a time that scales linearly with the number of particles, if the system satisfies area law.[11] While the proof of the area law for 2D (gapped) systems remains open, a vast body of evidence suggests that the conjecture is likely to be true. Even if area law turns out to be incorrect for general gapped systems, our result can be still applied to any systems that satisfy area law, which includes virtually all the known gapped systems in two spatial dimensions. Interestingly, our method is even applicable to systems that can host anyons, thus demonstrating its generality.

We believe this result is interesting in its own right, since it substantially generalizes the results of Ref.8–10. However, we also believe that the novel approach taken in this paper is likely to be useful for researachers working in other areas of quantum information science as well. This is primarily due to the fact that our work is based upon a structure of approximately conditionally independent states, which is new to the best of the author's knowledge. Typically a tripartite state is referred to be conditionally independent if its conditional mutual information, $I(A:C|B) = S(AB)+S(BC)-S(B)-S(ABC)$, is equal to 0.[14] Such states form a quantum Markov chain, which means that the correlation between A and C is completely mediated by B.[12] Approximately conditionally independend states refer to tripartite states whose conditional mutual information is close to, but not equal to 0. While approximately conditionally independent classical states are known to form an approximate Markov chain, the same statement is known to be false for general quantum states.[13] Due to the obstructions discussed in Ref.13, the structure of approximately conditionally independent states have remained elusive.

Our work shows that approximately conditionally independent states are locally defined. That is, if there are two approximately conditionally independent states($I(A:C|B)\approx 0$) that are locally consistent(over AB and BC), they are globally consistent as well. Therefore, if one is given a promise that the two states are approximately conditionally independent, it suffices to check their equivalence over local subsystems in order to certify their global equivalence. This method can be used recursively to certify an equivalence between two different multipartite states by checking their equivalence over a number of subsystems that grows only linearly with the system size.

To summarize, our work provides an efficient method to check the equivalence between two many-body quantum states, a task that is likely to be a common routine for quantum simulators in the future. The method covers the range of states that lie outside the scope of the previous works, two-dimensional gapped quantum many-body systems being the primary example. The idea behind the method is based on a simple yet general observation on the structure of approximately conditionally independent states, which may be of an interest to the wider community.

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[14] Here $S(AB)$ is the entropy of a composite system AB, and the other quantitites are defined similarly.