Single-shot fault-tolerant quantum error correction

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The development of efficient fault-tolerant quantum computing techniques is essential: they provide the means to deal with the decoherence and control imprecisions that are intrinsic to quantum systems, see *e.g.* [1]. Only the presence of such error sources prevents extending the ability to control small quantum systems for a limited time to that of performing arbitrarily long, precise and large quantum computations.

A key ingredient of fault-tolerance is the encoding of quantum information in non-local degrees of freedom that are not easily accessible to the environment. Local degrees of freedom can then absorb the damage caused by errors as the computation proceeds, but only up to a point. Thus errors need to be extracted to prevent (to a large extent) their catastrophic accumulation, which is achieved through *error correction* operations.

Naturally error correction is itself a noisy process: care has to be taken so that the attempt to eliminate errors does not end up introducing more. Typically error correction involves first an error detection process where the *error syndrome* is recovered, and errors at this stage can give rise to large, non-local effects in the correction stage. An approach to this problem [2] is to perform multiple times the measurements from which the error syndrome is to be inferred [3]. As a drawback, longer times and more operations lead themselves to more accumulation of errors.

This work [4] shows how certain codes are robust against imperfections in error detection: for them *a single round of local measurements* suffices. Local, because all the codes considered are local and in particular topological [5]. Namely, physical qubits are arranged in a lattice of variable size, the error syndrome can be recovered from local measurements, and logical operators have a support comparable to the lattice size.

The idea is, in essence, simple. One has to choose codes such that local errors in the recovery of the error syndrome will give rise to local errors at the correction stage. In practice things are a bit more tricky as in fact there exist flexibility in defining local noise, always preserving compatibility with the final goal of fault-tolerant quantum computing. It is by using this flexibility wisely that codes that have been known for more than a decade turn out to have this surprising single-shot property.

Connection with self-correction

Topological stabilizer codes can be regarded as ground states of gapped local Hamiltonians: they give (theoretical) examples of a form of quantum order in condensed matter, namely topological order [6]. Some of these topologically ordered phases are special in that they survive at finite temperatures. This means that the topological degeneracy of the ground state is not affected by thermal noise, so that such systems would provide a self-correcting quantum memory: a quantum memory that does not require active error correction.

Surprisingly, there exists a connection between self-correction and single-shot fault-tolerant error correction. Namely, all the known topological stabilizer codes that give rise to self-correcting systems turn out to have the single-shot property. The connection boils down to the fact that in both cases the connectivity of excitations (which correspond to the error syndrome) gives rise to their confinement.

Constant time overhead

Subsystem codes are those in which the code space contains both logical and gauge qubits. The latter are just qubits that we do not care about, and in a topological subsystem code they might include local degrees of freedom. Notably, unlike in conventional topological codes there is not such a clear-cut connection between topological subsystem codes and topological order. This is a subject that still requires further study.

Gauge color codes are a remarkable class of topological subsystem codes [7]. In particular, 3D gauge color codes allow the transversal implementation of a universal set of gauges up to gauge fixing [8], a technique that is closely connected to error correction. This is very interesting by itself, but in addition it turns out that making use of the subsystem degrees of freedom at the error detection stage allows to perform fault-tolerant quantum error correction with a single round of measurements (and the same holds for gauge fixing!).

Indeed, in the case of 3D gauge color codes all required operations for quantum computation (initialization, gates and measurements) can be performed fault-tolerantly with a local quantum circuit of finite depth assisted with non-local classical computations (that can be efficiently performed). This provides a rather singular approach to fault-tolerant quantum computing that should give rise to very interesting developments in the future, including practical applications.

It should be noted that all other known topological codes with the single-shot property have at least 4 spatial dimensions. Indeed, a fundamental open problem is whether a self-correcting topological phase might exist in 3D. In this regard, one can conjecture that a Hamiltonian system connected to 3D gauge color codes could give rise to a self-correcting phase.

References

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[3] Alternatively this effort can be transferred to the preparation of highly entangled states [**?**, **?**].

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